2.26.1. Valuation and Anti-Valuation Sentence Problems

A. Without using truth tables or truth trees, decide whether each of the following argument is **valid** or **invalid**.

(1)

1. $(P \lor Q)$
2. $(\sim P \lor \sim Q)$
3. $(\sim P \lor Q)$

$\therefore \ \sim(P \lor \sim Q)$

(2)

1. $(P \lor Q)$
2. $(P \lor \sim Q)$

$\therefore \ \sim(P \lor Q)$

(3)

1. $\sim(P \lor Q)$

$\therefore \ (\sim P \lor \sim Q)$

(4)

1. $\sim(P \lor Q)$

$\therefore \ (P \lor Q)$

B. For each of the following sentences, state whether it is a **tautology**, a **contradiction**, or **neither**.

1. $(\sim P \land \sim Q) \land (\sim P \land Q)$
2. $(\sim P \lor \sim Q) \lor (\sim P \lor Q)$
3. $(\sim P \land \sim Q) \lor (\sim P \land Q)$
4. $(\sim P \land \sim Q) \lor (\sim P \land Q) \lor (P \land Q)$
5. $(\sim P \land \sim Q) \lor (\sim P \land Q) \lor (P \land \sim Q) \lor (P \land Q)$
6. $(\sim P \lor \sim Q) \land (\sim P \lor Q) \land (P \lor \sim Q) \land (P \lor Q)$
7. $(\sim P \lor \sim Q) \land (P \lor \sim Q) \land (\sim P \lor Q)$

C. For each of the following sentences, state a **simpler sentence** which is **logically equivalent** to that sentence. (*Don't use truth tables.*)

1. $(\sim P \land \sim Q) \lor (\sim P \land Q) \lor (P \land Q)$
2. $(P \lor \sim Q) \land (\sim P \lor Q) \land (\sim P \lor \sim Q)$
3. $(P \lor Q) \land (\sim P \lor Q) \land (P \lor \sim Q)$

D. We noted that in a family of anti-valuation sentences, the negation of one follows from the other sentences in that family – as in the following example.

$$\begin{array}{c} (\sim P \vee Q) \\ (P \vee \sim Q) \\ (\sim P \vee \sim Q) \\ \hline \therefore\ \sim(P \vee Q) \end{array}$$

But if one of those premises is left out, the resulting argument is invalid.

$$\begin{array}{c} (\sim P \vee Q) \\ (P \vee \sim Q) \\ \hline \therefore\ \sim(P \vee Q) \end{array}$$

Which needed premise is missing from the above invalid argument? Which truth table valuation is a validity counterexample for this argument? Is the missing premise true or false in that valuation? If two premises had been missing, how many counterexamples would the resulting argument suffer from?

1. Guided by answers to these questions, state a general method for telling (without building truth tables) what the validity counterexample(s) for such an argument are, based on the missing premise(s) from that family. Use this method to state (without building truth tables) what the counterexample(s) will be for the following invalid arguments.

$$\begin{array}{c} (P \vee Q) \\ (\sim P \vee Q) \\ \hline \therefore\ \sim(\sim P \vee \sim Q) \end{array}$$

$$\begin{array}{c} (P \vee Q) \\ \hline \therefore\ \sim(\sim P \vee \sim Q) \end{array}$$

$$\begin{array}{c} ((P \vee Q) \vee R) \\ ((\sim P \vee Q) \vee R) \\ ((\sim P \vee \sim Q) \vee R) \\ ((P \vee Q) \vee \sim R) \\ ((\sim P \vee Q) \vee \sim R) \\ \hline \therefore\ \sim((\sim P \vee \sim Q) \vee \sim R) \end{array}$$

2. Work out a similar procedure for arguments using valuation sentences and their negations.